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Logarithmic two dimensional spin-1/3 fractional supersymmetric conformal field theories and the two-point functions

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Abstract. Logarithmic spin-1/3 superconformal field theories are investigated. The chiral and full two-point functions of two- (or higher-) dimensional Jordanian blocks of arbitrary weights are obtained.

1 Introduction

According to Gurarie [1], conformal field theories of which the correlation functions exhibit logarithmic behaviour may be consistently defined. In some interesting physical theories like polymers [2], WZNW models [3–6], percolation [7], the Haldane–Rezayi quantum Hall state [8], and edge excitation in the fractional quantum Hall effect [9], logarithmic correlation functions appear. Also the logarithmic operators can be considered in 2D-magnetohydrodynamic turbulence [10–12], 2D-turbulence ([13, 14]) and some critical disordered models [15,16]. Logarithmic conformal field theories for the D-dimensional case ($D > 2$) have also been studied [17]. In this paper we consider a superconformal extension of the Virasoro algebra [18,19] corresponding to three-component supermultiplets, and then, following [20–22], generalize the superconformal field to Jordanian blocks of quasi-superconformal fields. We then find the two-point functions of chiral- and full-component fields. It is seen that these correlators are readily obtained through formal derivatives of correlators of superprimary fields, just as was seen in [20–22].

2 Superprimary and quasi-superprimary fields

A chiral superprimary field $\Phi(z,\theta)$ with conformal weight Δ is an operator satisfying [18]

$$[L_n,\Phi(z,\theta)] = \left[z^{n+1}\partial_z + (n+1)\left(\Delta+\frac{\Lambda}{3}\right)z^n\right] \times \Phi(z,\theta), \quad (1)$$

$$[G_r,\Phi(z,\theta)] = [z^{r+1/3}\delta_\theta - q^2 z^{r+1/3}\theta^2\partial_z - (3r+1)q^2\Delta z^{r-2/3}\theta^2]\Phi(z,\theta), \quad (2)$$

where θ is a para-Grassmann variable, satisfying $\theta^3=0$, q is one of the third roots of unity, not equal to one, and δ_θ and Λ satisfy [18,19]

$$\delta_\theta\theta = q^{-1}\theta\delta_\theta + 1, \quad (3)$$

and

$$[\Lambda,\theta]=\theta, \quad [\Lambda,\delta_\theta] = -\delta_\theta. \quad (4)$$

Here L_n's and G_r's are the generators of the super-Virasoro algebra satisfying

$$[L_n,L_m] = (n-m)L_{n+m}, \qquad [L_n,G_r] = \left(\frac{n}{3}-r\right)G_{n+r}, \quad (5)$$

and

$$G_rG_sG_t + \text{ five other permutations of the indices} = 6L_{r+s+t}. \quad (6)$$

The superprimary field $\Phi(z,\theta)$ is written

$$\Phi := \Phi(z,\theta) = \varphi(z,\theta) + \theta\varphi_1(z) + \theta^2\varphi_2(z), \quad (7)$$

where $\varphi_1(z)$ and $\varphi_2(z)$ are para-Grassmann fields of grades 1 and 2, respectively. One can similarly define a complete superprimary field $\Phi(z,\bar z,\theta,\bar\theta)$ with the weights $(\Delta,\bar\Delta)$ and the expansion

$$\Phi = \sum_{k,k'=0}^{2}\theta^k\bar\theta^{k'}\varphi_{kk'}, \quad (8)$$

through (1) and (2), and obvious analogous relations with the $\bar L_n$'s and $\bar G_r$'s. Now suppose that the first component field $\varphi(z)$ in the chiral superprimary field $\Phi(z,\theta)$ has a logarithmic counterpart $\varphi'(z)$ [20]:

$$[L_n,\varphi'(z)] = [z^{n+1}\partial_z + (n+1)z^n\Delta]\varphi'(z) + (n+1)z^n\varphi(z). \quad (9)$$

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We will show that $\varphi'(z)$ is the first component field of a new superfield $\Phi'(z,\theta)$, which is the formal derivative of the superfield $\Phi(z,\theta)$ with respect to its weight. Let us define the fields $f'_r(z)$ by

$$[G_r, \varphi'(z)] =: z^{r+1/3} f'_r(z), \tag{10}$$

where $r + (1/3)$ is an integer. Following [22], acting on both sides of the above equation with L_m and using the Jacobi identity, and using (9), (1), and (5), we have

$$\begin{aligned}[L_m, f'_r(z)] &= \left(\frac{m}{3} - r\right) z^m [f'_{m+r}(z) - f'_r(z)] \\ &+ \left[z^{m+1}\partial_z + (m+1)\left(\Delta + \frac{1}{3}\right) z^m\right] f'_r(z) \\ &+ (m+1) z^m \varphi_1(z).\end{aligned} \tag{11}$$

Demanding

$$[L_{-1}, f'_r(z)] = \partial_z f'_r(z), \tag{12}$$

it is easy to show that

$$f'_r(z) = \begin{cases} \psi'(z), & r \geq -1/3, \\ \psi''(z), & r \leq -4/3. \end{cases} \tag{13}$$

Then, equating $[L_1, f'_{-4/3}(z)]$ and $[L_1, f'_{-7/3}(z)]$, we obtain

$$\psi'(z) = \psi''(z) =: \psi'_1. \tag{14}$$

So in this way we obtain a well-defined field ψ'_1, satisfying

$$[G_r, \varphi'] = z^{r+1/3} \psi'_1, \tag{15}$$

$$\begin{aligned}[L_n, \psi'_1] &= \left[z^{n+1}\partial_z + (n+1) z^n \left(\Delta + \frac{1}{3}\right)\right] \psi'_1 \\ &+ (n+1) z^n \psi_1.\end{aligned} \tag{16}$$

Again, let us define the fields $h'_r(z)$ through

$$[G_r, \psi'_1]_{q^{-1}} := -z^{r+1/3} h'_r(z). \tag{17}$$

Acting on both sides with L_m and using the generalized Jacobi identity [18]:

$$\begin{aligned}&[[G_r, \psi'_1]_{q^{-1}}, L_m] + [G_r, [L_m, \psi'_1]]_{q^{-1}} \\ &+ [[L_m, G_r], \psi'_1]_{q^{-1}} = 0,\end{aligned} \tag{18}$$

we obtain

$$\begin{aligned}[L_m, h'_r] &= \left[z^{m+1}\partial_z + (m+1)\left(\Delta + \frac{1}{3}\right) z^m\right] h'_r \\ &+ (m+1) z^m \psi_2 + \left(\frac{m}{3} - r\right) z^m h'_{m+r} \\ &+ \left(r + \frac{1}{3}\right) z^m h'_r.\end{aligned} \tag{19}$$

Then, using the same method applied to determine the form of the functions $f'_r(z)$, we find a well-defined field ψ'_2 satisfying

$$[G_r, \psi'_1]_{q^{-1}} = -z^{r+1/3} \psi'_2, \tag{20}$$

$$\begin{aligned}[L_n, \psi'_2] &= \left[z^{n+1}\partial_z + (n+1)\left(\Delta + \frac{2}{3}\right) z^n\right] \psi'_2 \\ &+ (n+1) z^n \psi_2.\end{aligned} \tag{21}$$

Finally, we must calculate $[G_r, \psi'_2]_{q^{-2}}$. Substituting for $\psi'_2(z)$ from (20) and (15), and using (6), we have

$$\begin{aligned}[G_r, \psi'_2]_q &= -[z^{r+1/3}\partial_z \varphi'(z) + (3r+1) z^{r-2/3} \Delta \varphi'(z) \\ &+ (3r+1) z^{r-2/3} \varphi(z)].\end{aligned} \tag{22}$$

Now we define the quasi-superprimary field Φ':

$$\Phi' := \Phi'(z,\theta) = \varphi'(z) + \theta \psi'_1(z) + \theta^2 \psi'_2(z). \tag{23}$$

It is easy to see that

$$\begin{aligned}[L_n, \Phi'] &= \left[z^{n+1}\partial_z + (n+1) z^n \left(\Delta + \frac{\Lambda}{3}\right)\right] \Phi' \\ &+ (n+1) z^n \Phi,\end{aligned} \tag{24}$$

$$\begin{aligned}[G_r, \Phi'] &= [z^{r+1/3}(\delta_\theta - q^2\theta^2\partial_z) - (3r+1) z^{r-2/3} q^2 \Delta \theta^2] \Phi' \\ &- q^2 (3r+1) z^{r-2/3} \theta^2 \Phi.\end{aligned} \tag{25}$$

We see that (24) and (25) are formal derivatives of (1) and (2) with respect to Δ, provided one defines the formal derivative [20–22]

$$\Phi'(z,\theta) =: \frac{\mathrm{d}\Phi}{\mathrm{d}\Delta}. \tag{26}$$

The two superfields Φ and Φ', are a two-dimensional Jordanian block of quasi-primary fields. The generalization of the above results to an m-dimensional Jordanian block is obvious:

$$\begin{aligned}[L_n, \Phi^i] &= \left[z^{n+1}\partial_z + (n+1) z^n \left(\Delta + \frac{\Lambda}{3}\right)\right] \Phi^i \\ &+ (n+1) z^n \Phi^{i-1},\end{aligned} \tag{27}$$

and

$$\begin{aligned}[G_r, \Phi^{(i)}] &= [z^{r+1/3}(\delta_\theta - q^2\theta^2\partial_z) \\ &- (3r+1) q^2 z^{r-2/3} \Delta \theta^2] \Phi^i \\ &- q^2 \theta^2 (3r+1) z^{r-2/3} \Phi^{(i-1)}.\end{aligned} \tag{28}$$

Here $1 \leq i \leq m-1$, and the first member of the block, $\Phi^{(0)}$, is a superprimary field. It is easy to show that (27) and (28) are satisfied through the formal relation

$$\Phi^{(i)} = \frac{1}{\mathrm{i}!} \frac{\mathrm{d}^i \Phi^{(0)}}{\mathrm{d}\Delta^i}. \tag{29}$$

3 Two-point functions of Jordanian blocks

Consider two Jordanian blocks of chiral quasi-primary fields Φ_1 and Φ_2, with the weights Δ_1 and Δ_2 and dimensions p and q, respectively. As the only closed subalgebra of the super-Virasoro algebra of which the central extension is trivial is formed by $[L_{-1}, L_0, G_{-1/3}]$, the correlator of fields with different weights may be nonzero. According to [18],

$$\langle\varphi_k^{(0)}\varphi_{k'}^{\prime(0)}\rangle = a_K \frac{A_{kk'}(\Delta+\Delta'+(k+k'-3)/3)^{B_{kk'}}}{(z-z')^{\Delta+\Delta'+(k+k')/3}} =: a_K f_{k,k'}(z-z'), \tag{30}$$

where $A_{kk'}$ and $B_{kk'}$ are the components of the following matrices:

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ q^2 & -q^2 & q^2 \end{pmatrix}, \tag{31}$$

a_0, a_1, and a_2, are arbitrary constants, and

$$K = k + k' \bmod 3. \tag{32}$$

The general form of the two-point functions of Jordanian blocks is then readily obtained, using (29):

$$\langle\varphi_k^{(i)}\varphi_{k'}^{\prime(j)}\rangle = \frac{1}{i!}\frac{1}{j!}\frac{\mathrm{d}^i}{\mathrm{d}\Delta^i}\frac{\mathrm{d}^j}{\mathrm{d}\Delta'^j} \times \frac{a_K A_{kk'}(\Delta+\Delta'+(k+k'-3)/3)^{B_{kk'}}}{(z-z')^{\Delta+\Delta'+(k+k')/3}}. \tag{33}$$

Here $0 \le i \le p-1$, and $0 \le j \le q-1$. In this formal differentiation, one should treat the constants a_i as functions of Δ and Δ'. So there will be other arbitrary constants

$$a_i^{(j),(k)} := \frac{\mathrm{d}^j}{\mathrm{d}\Delta^j}\frac{\mathrm{d}^k}{\mathrm{d}\Delta^k} a_i \tag{34}$$

in these correlators.

To consider the correlators of the full field, one begins with

$$\langle\varphi_{k\bar{k}}^{(00)}(z,\bar{z})\varphi_{k'\bar{k}'}^{\prime(00)}(z,\bar{z})\rangle = a_{K\bar{K}} q^{-k\bar{k}} f_{k,k'}(z-z')\bar{f}_{\bar{k},\bar{k}'}(\bar{z}-\bar{z}'), \tag{35}$$

obtained in [18]. Here $f_{k,k'}(z-z')$ is defined in (30) and $\bar{f}_{\bar{k},\bar{k}'}(\bar{z}-\bar{z}')$ is the same as this expression with $\Delta \to \bar{\Delta}$ and $\Delta' \to \bar{\Delta}'$. Also,

$$K = k + k' \bmod 3, \tag{36}$$

$$\bar{K} = \bar{k} + \bar{k}' \bmod 3. \tag{37}$$

Using the obvious generalization of (29), it is easy to see that

$$\langle\varphi_{k,\bar{k}}^{(ij)}\varphi_{k',\bar{k}'}^{(lm)}\rangle = \frac{1}{i!j!l!m!}\frac{\mathrm{d}^i}{\mathrm{d}\Delta^i}\frac{\mathrm{d}^j}{\mathrm{d}\bar{\Delta}^j}\frac{\mathrm{d}^l}{\mathrm{d}\Delta'^l}\frac{\mathrm{d}^m}{\mathrm{d}\bar{\Delta}'^m} \times [a_{K\bar{K}} f_{k,k'}\bar{f}\bar{k},\bar{k}']. \tag{38}$$

Again, one should treat the $a_{K\bar{K}}$'s as formal functions of the weights, so that differentiating them with respect to the weights introduces new arbitrary parameters.

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